# A Note on the Relationship between Different Types of Correction Queries

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# Outline

1 Introduction

2 Learning with Correction Queries

3 Polynomial Time Learning with Correction Queries

# Motivation

- The adult-child interaction has been the inspiration for Angluin's query learning model [Angluin87], the forerunner of today's active learning field.
- Drawbacks:
  - EQs are both unrealistic and computationally costly
  - MQs are not informative enough (no feedback)
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# Types of CQ

- Prefix Correction Queries (PCQs) [BeDeTi06]

$$C_L(u) = \begin{cases} \min\{v \mid uv \in L\}, & \text{if } u \in \mathit{Pref}(L) \\ \Theta, & \text{otherwise.} \end{cases}$$

- Length Bounded Correction Queries (LBCQs) [Tirn07]

$$C_L^l(u) = \{v \in \Sigma^* \mid uv \in L, |v| \leq l\}.$$

- Edit Distance Correction Queries (EDCQs) [BeHiJaTa07]

$$\textsc{Edc}_L(u) = \begin{cases} \text{Yes}, & \text{if } u \in L \\ v \in L \text{ s.t. } d(u,v) \text{ is minimum}, & \text{otherwise.} \end{cases}$$

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# Learning Models

- Concept: indexable class of recursive languages
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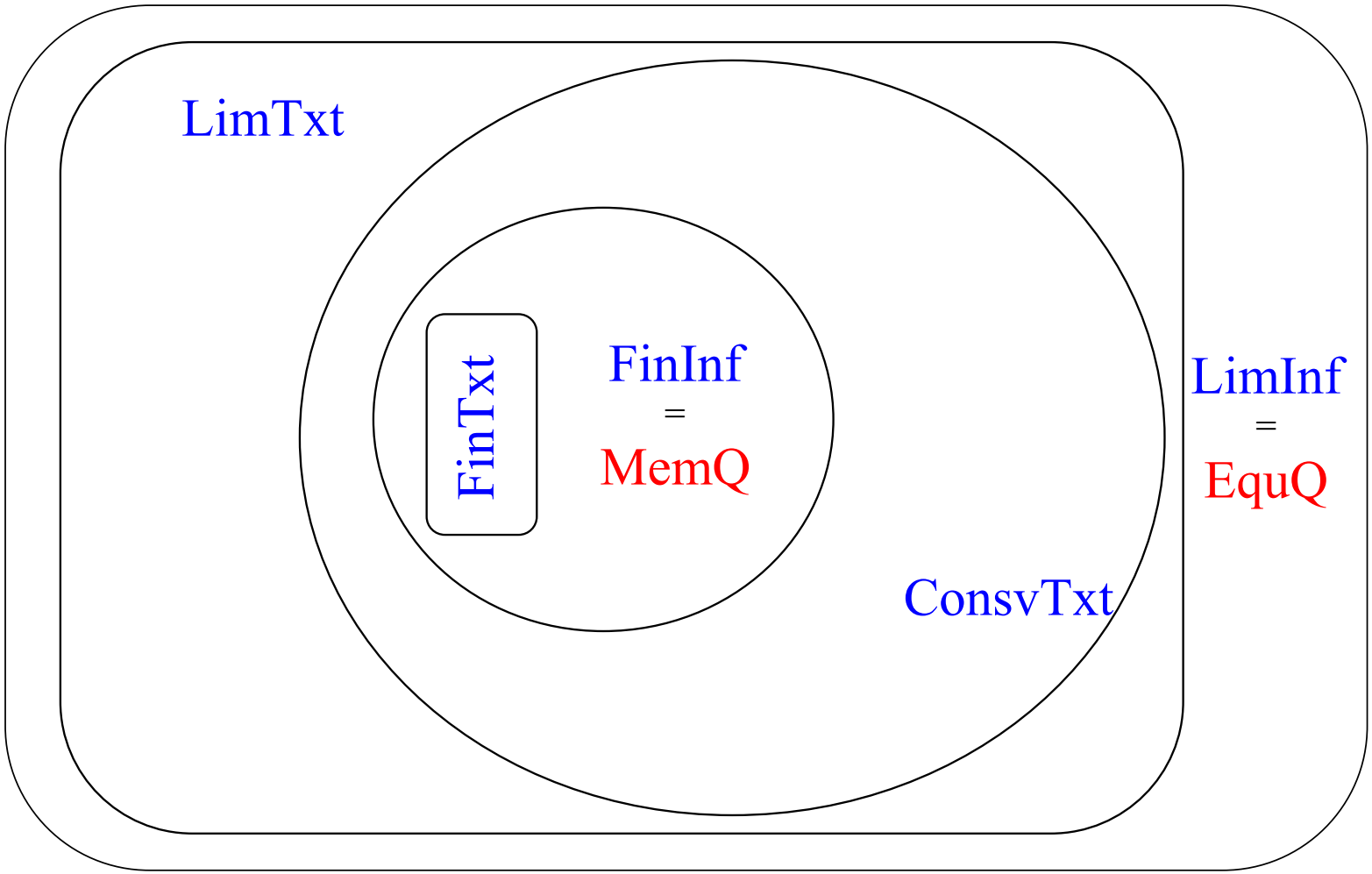
# Hierarchy [LanZil04]



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# Learning with PCQs [TirKob07]

**Definition (PCorQ)**

*all indexable classes $\mathcal{C}$ for which there is a query learner that identifies any language in $\mathcal{C}$ using a finite number of PCQs.*

- MemQ is strictly included in PCorQ,
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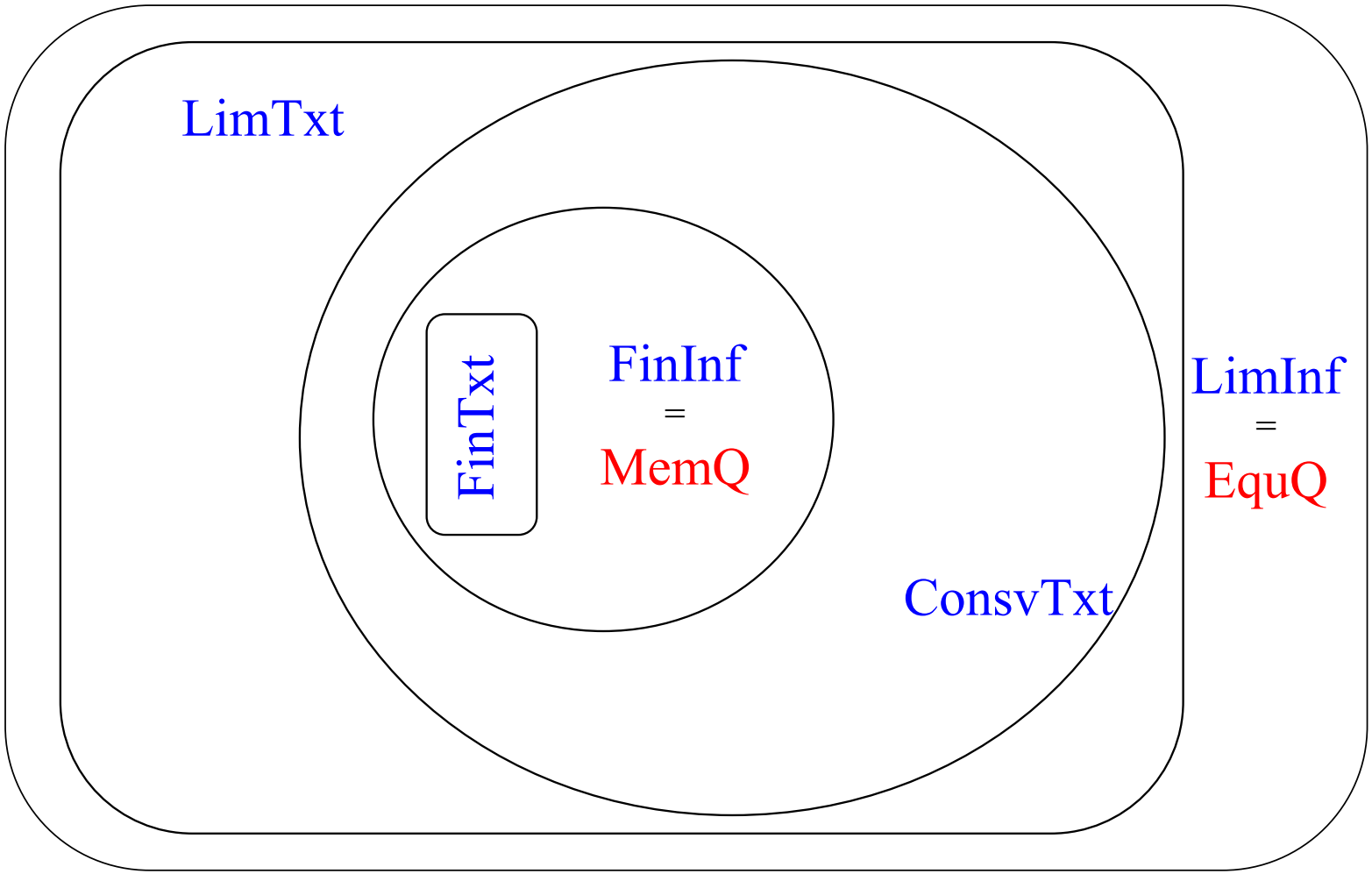

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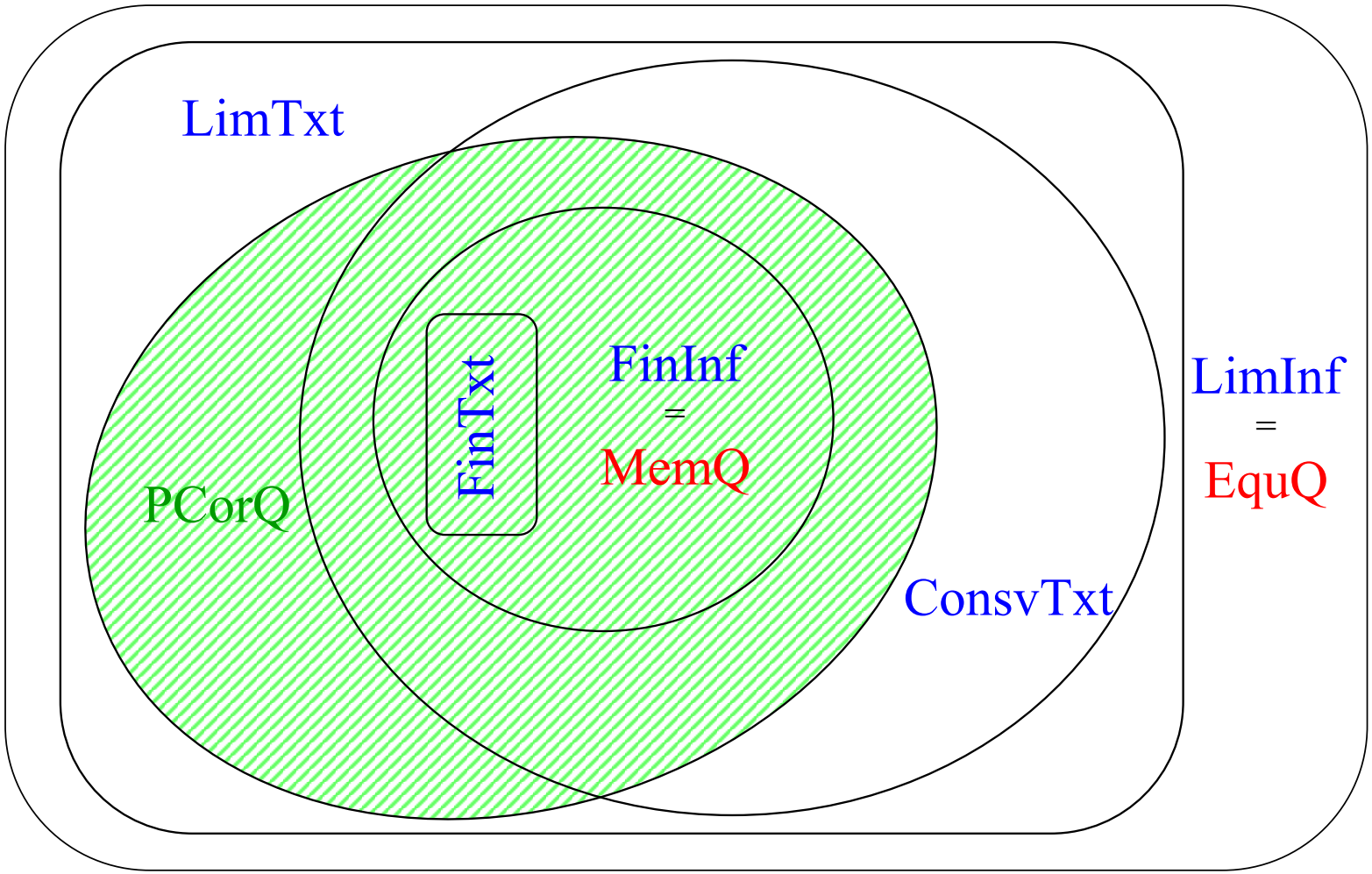

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*all indexable classes $\mathcal{C}$ for which there is a query learner that identifies any language in $\mathcal{C}$ using a finite number of $l$-bounded CQs.*

**Theorem**

*For any $l \geq 0$, IBCorQ = MemQ.*

**Corollary**

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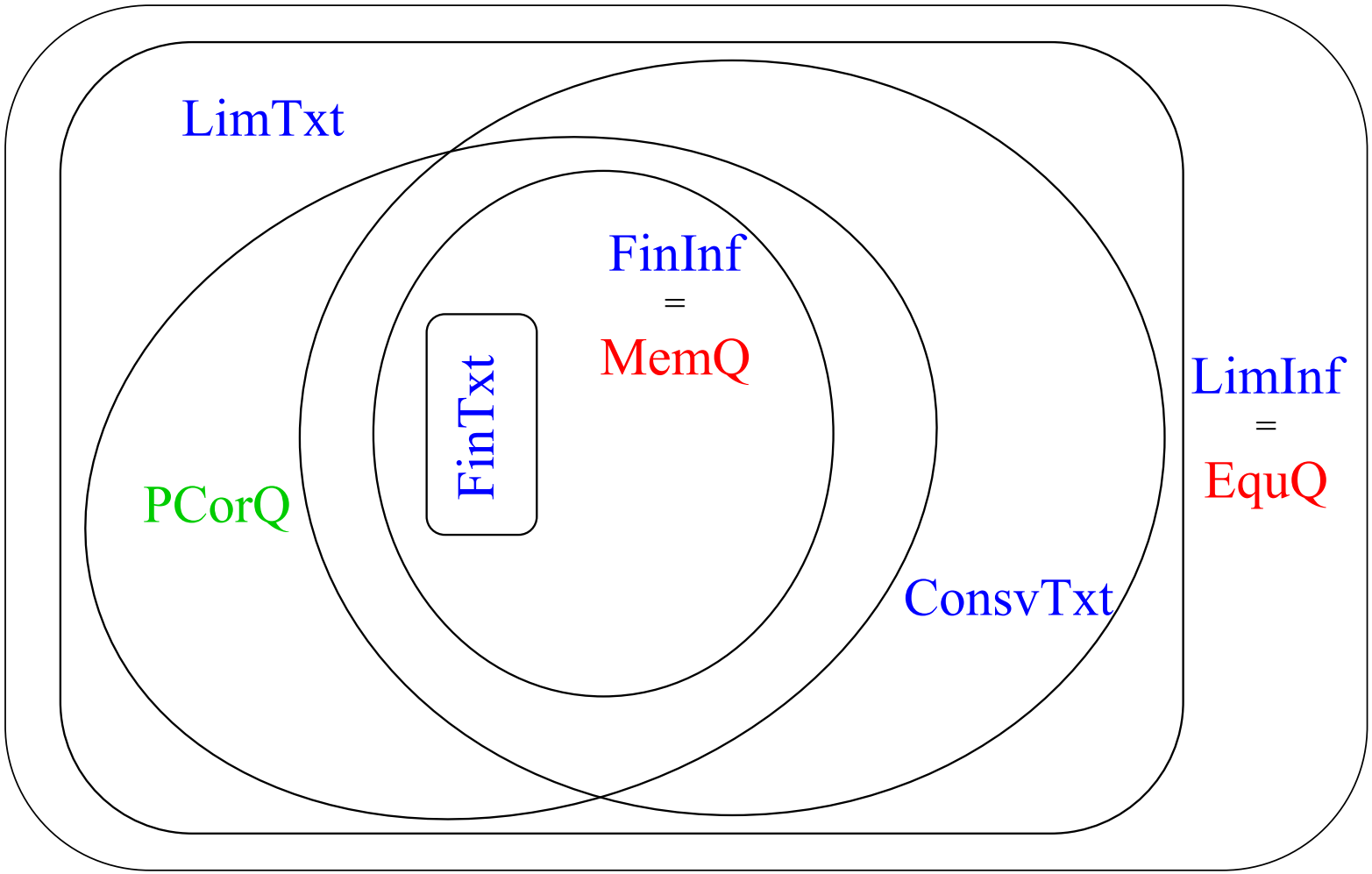

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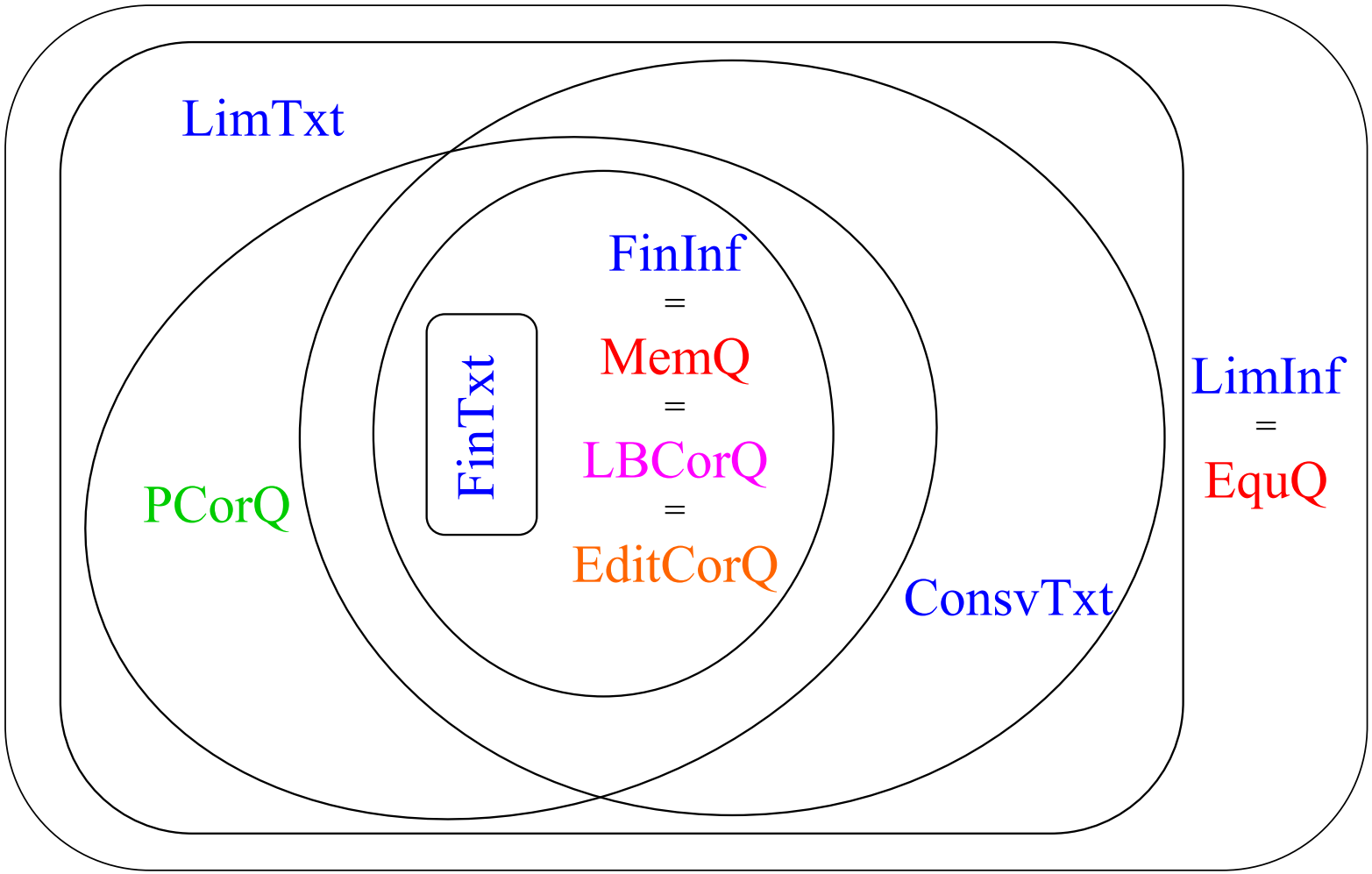

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| $MemQ \subsetneq PCorQ$ | $PolMemQ \subsetneq PolPCorQ$ [1] |
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**Proposition**

*Pol(l-1)BCorQ* $=$ *PolIBCorQ for any l* $\geq 1$.

**Remark**

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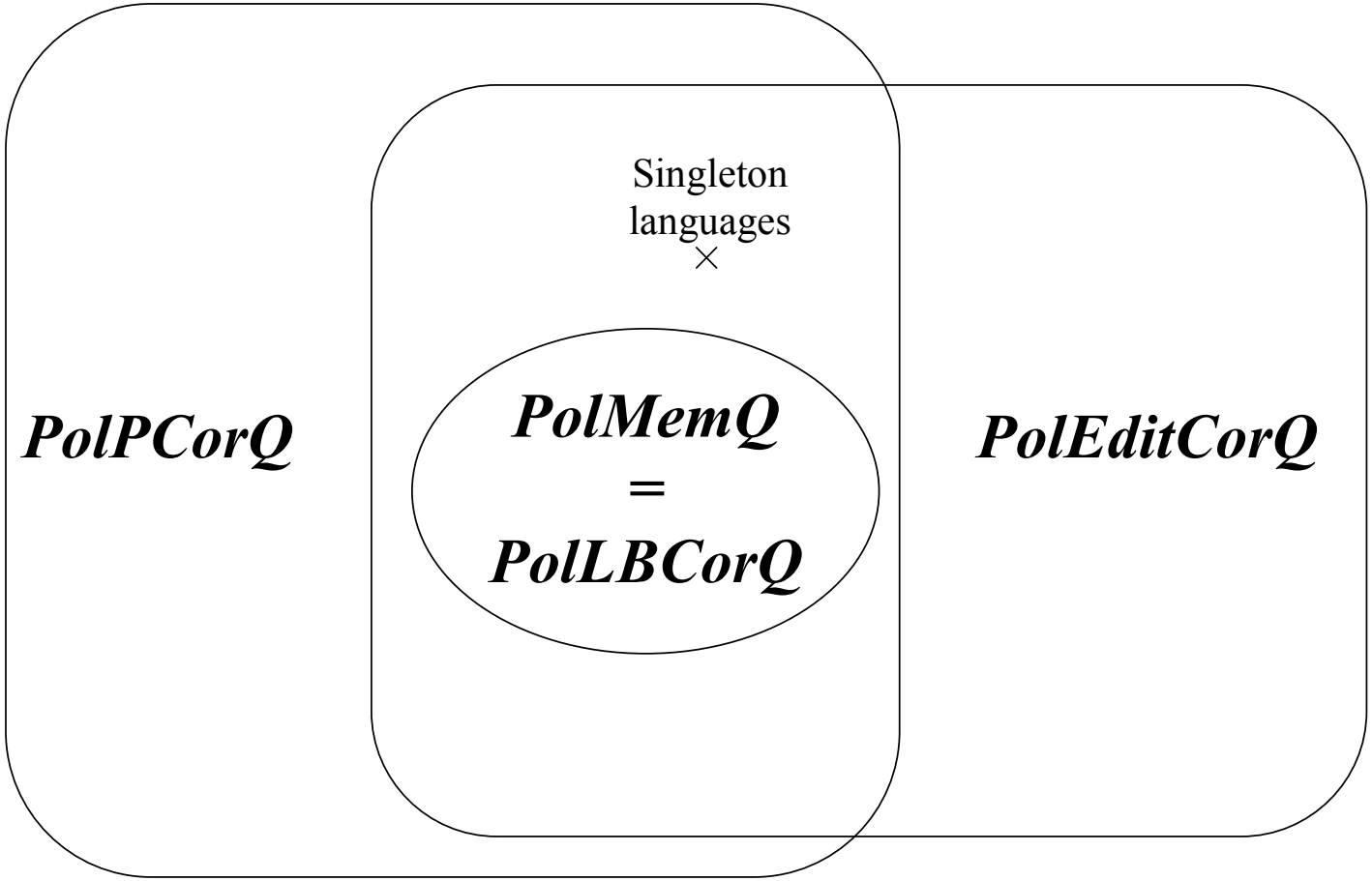

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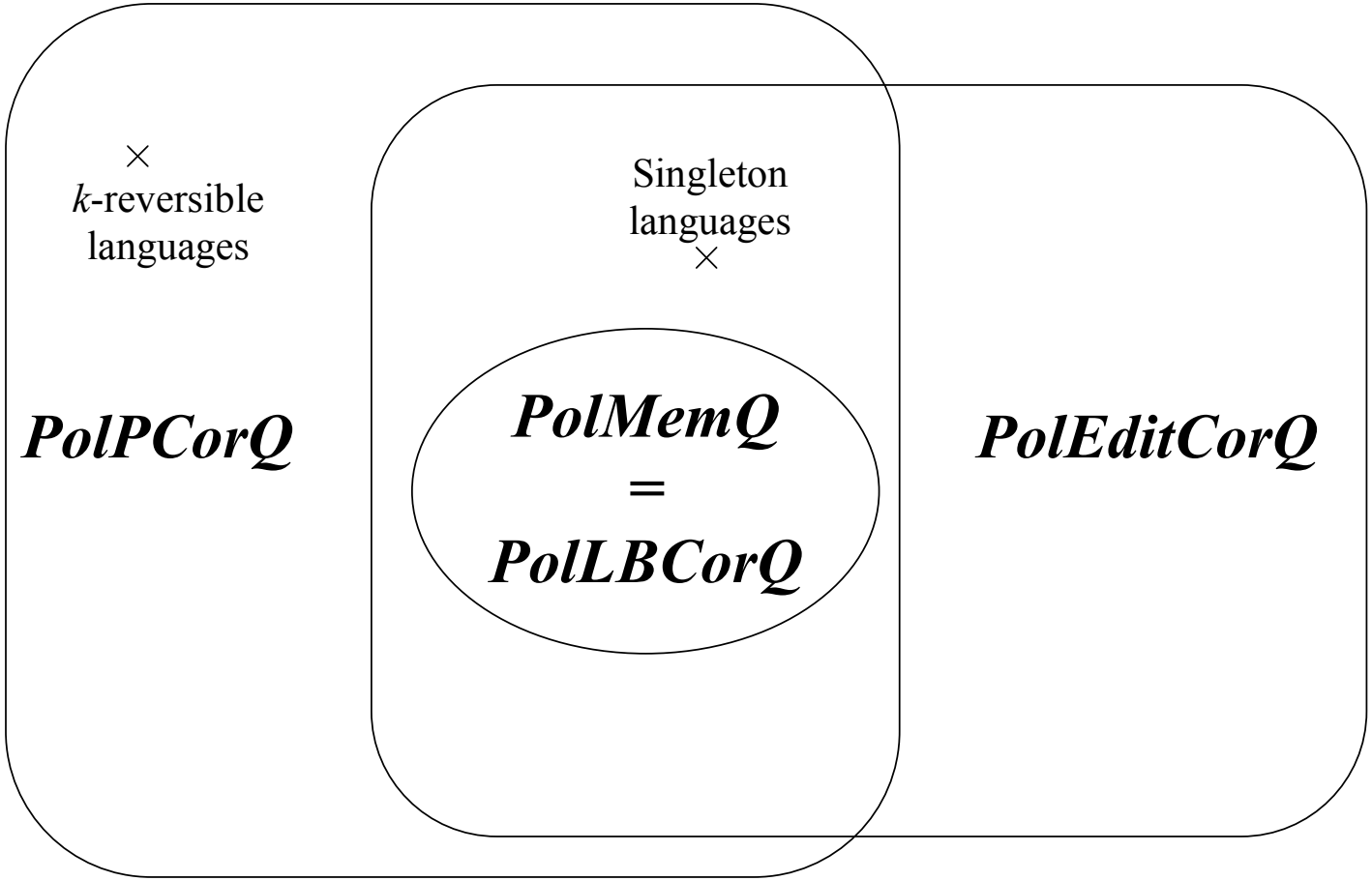

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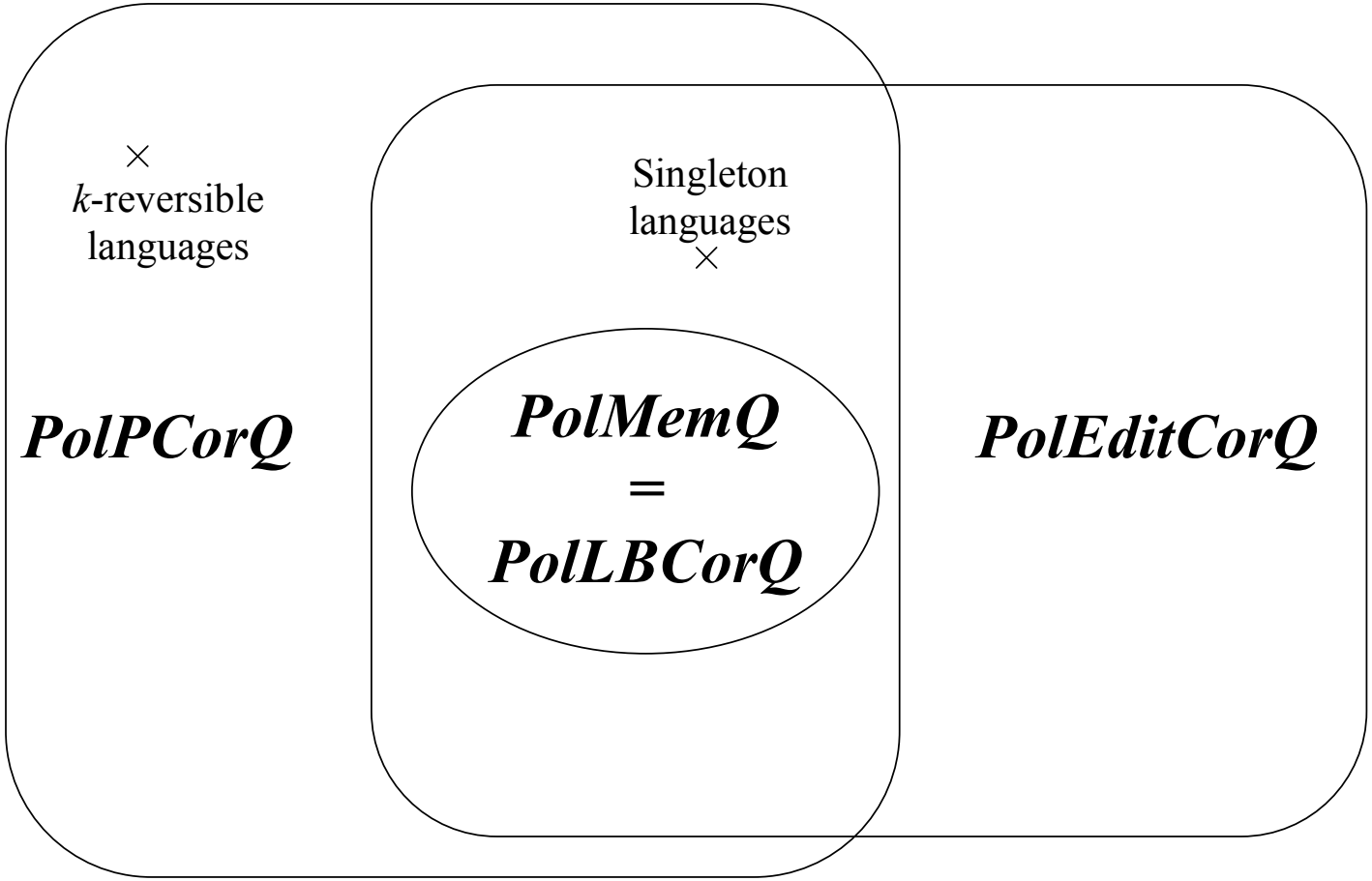


$k$-**Rev** $\notin$ *MemQ* [TirKnu07]

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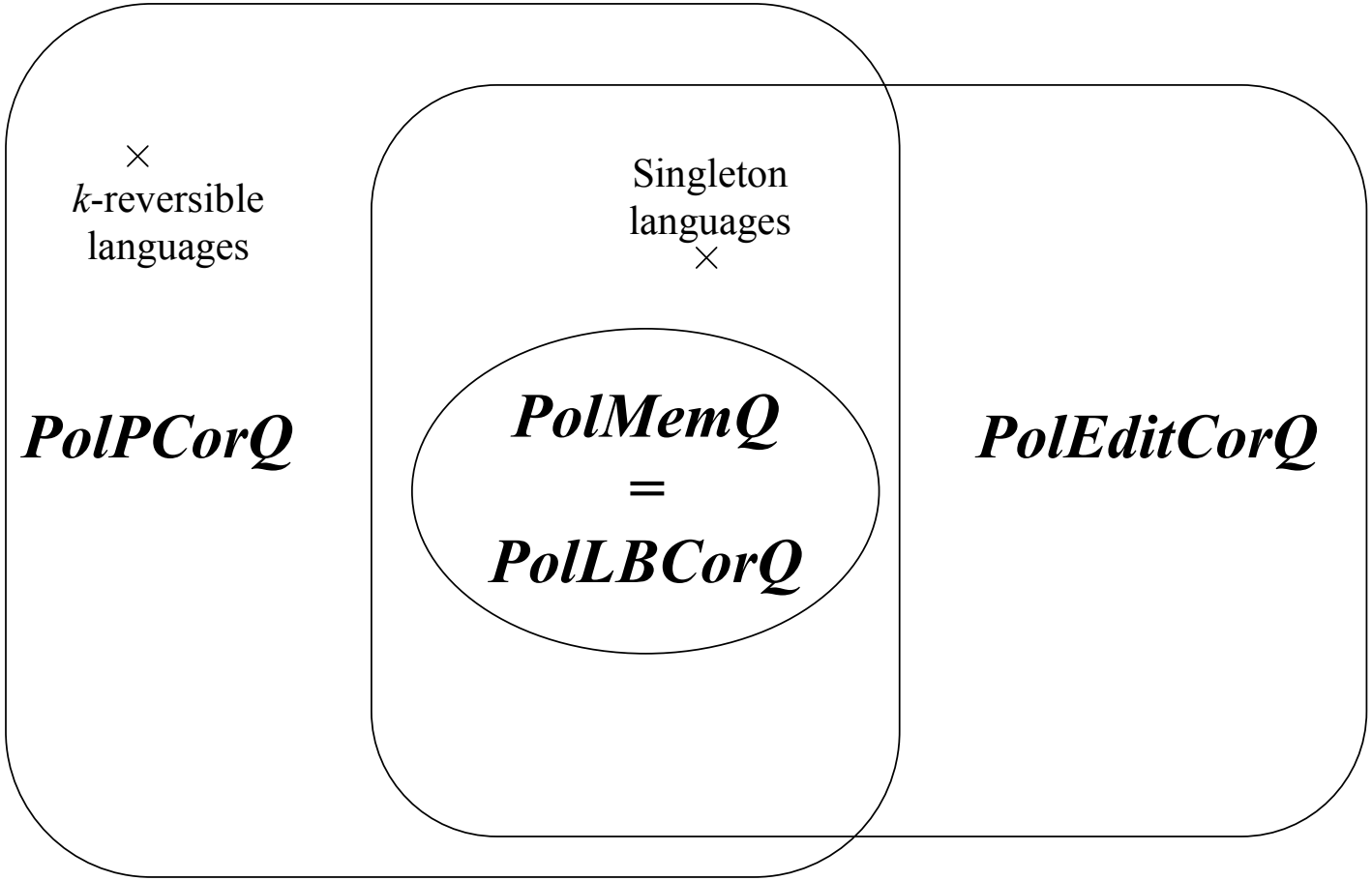


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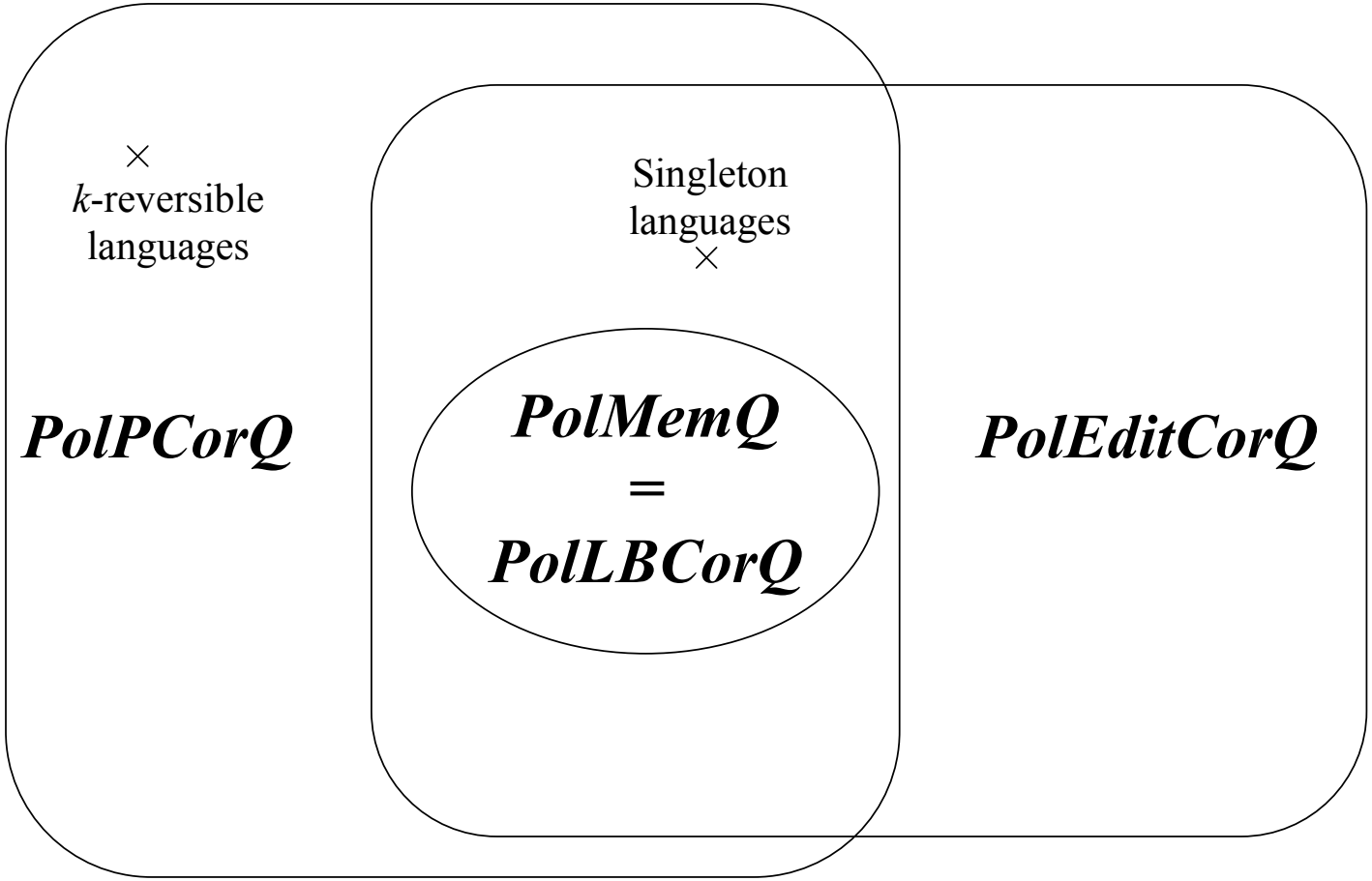


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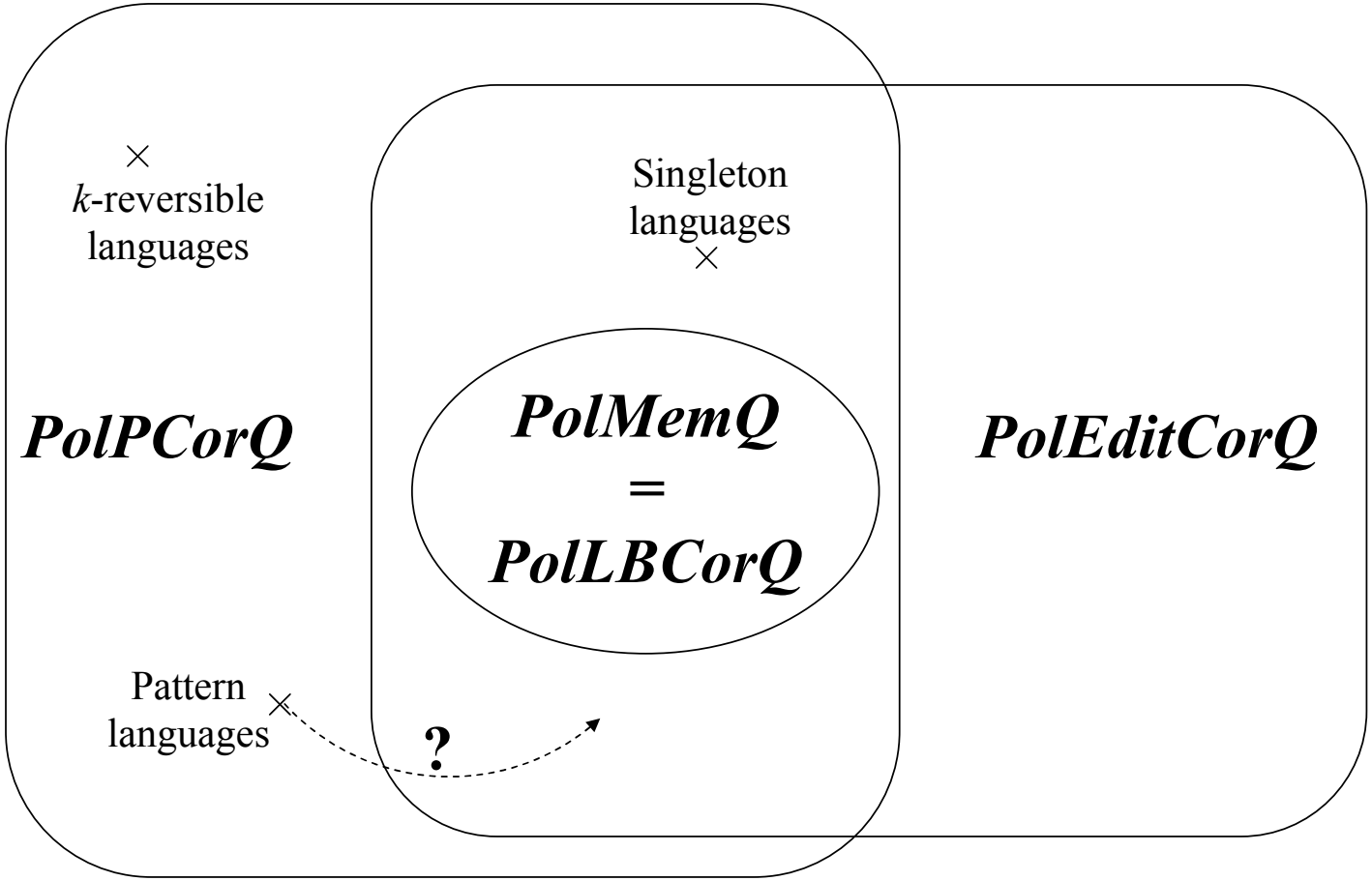

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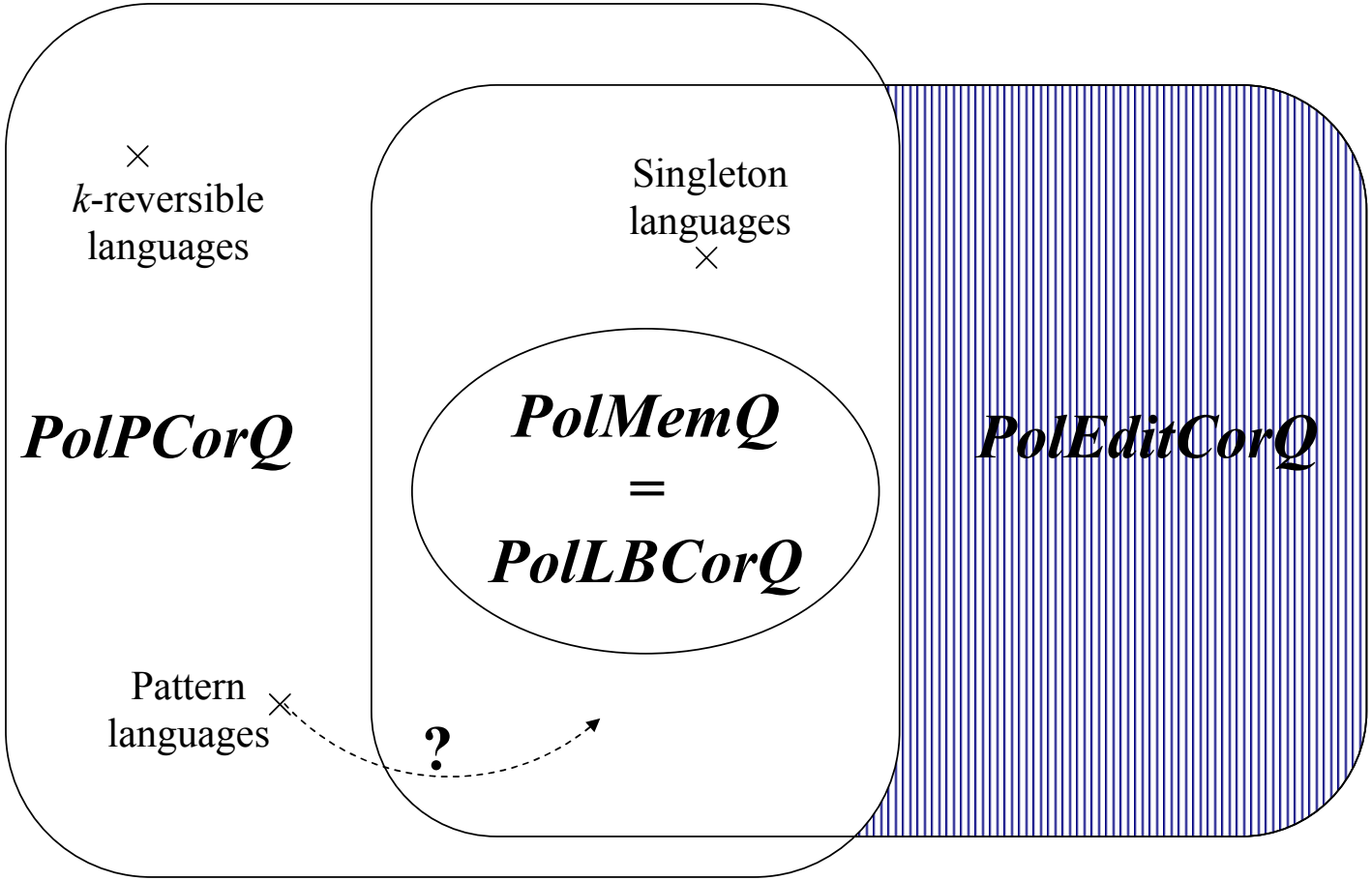

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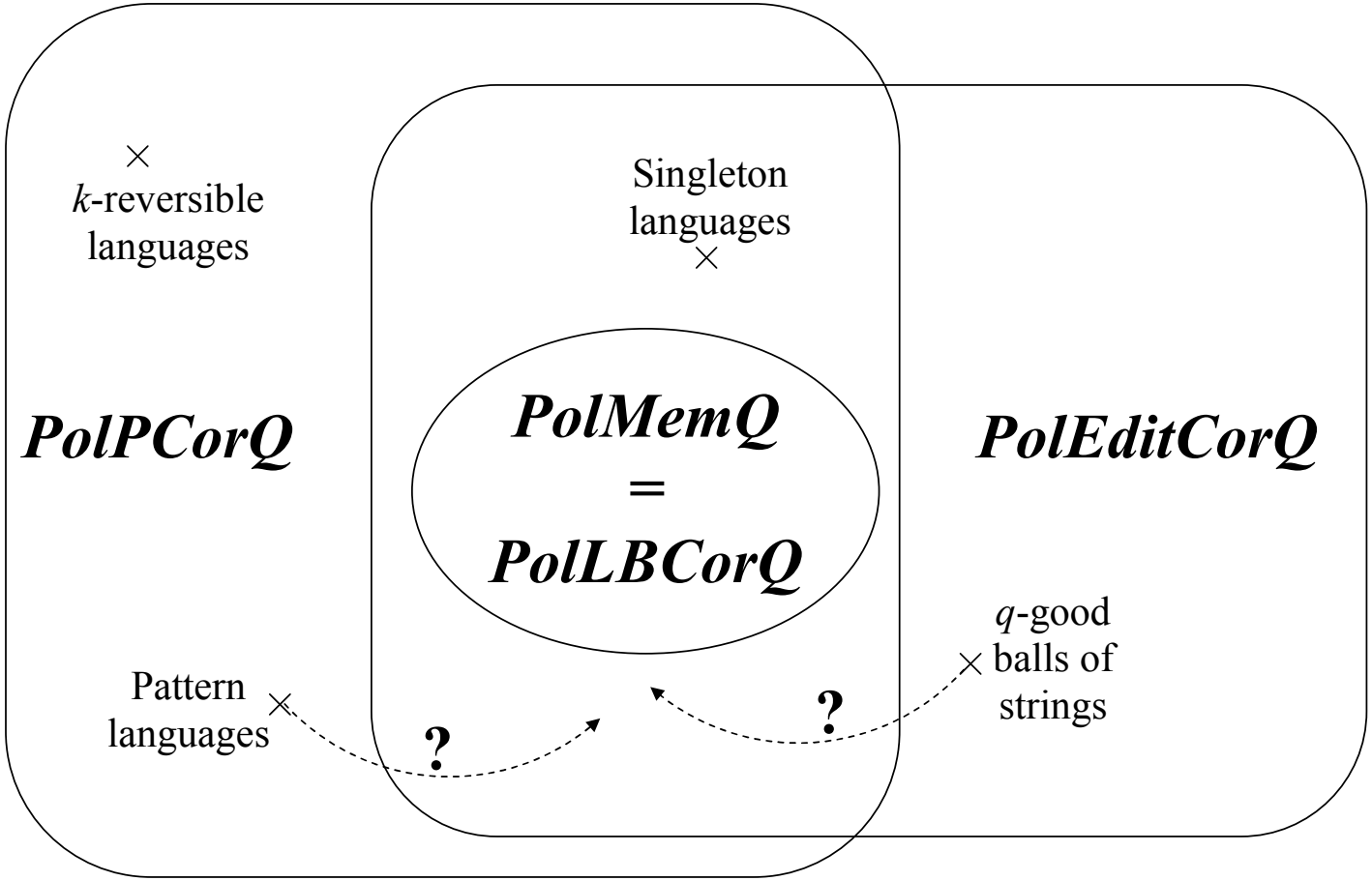

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Thank You!

Merci Beaucoup!

!Muchas Gracias!

Mulţumesc!