ICGI 2008:

A Learning Algorithm for Multi-dimensional Trees, or: Learning Beyond Context-Freeness

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- Part I – Formal groundwork: Multi-dimensional trees
- Part II – The adapted MAT learning algorithm L^* for multi-dimensional trees

Introduction

- It is possible to learn cf string languages via their connection (yield) to regular tree languages.
- We would like to show that this MAT-learnable class extends even further.
- We do that via *multi-dimensional trees* as structural descriptions.
- In order to obtain that result we have to introduce a new term-like notation for multi-dimensional trees which establishes them as a *direct and natural generalization* of classical trees.

Preliminaries – Trees

- The set T_Σ of all trees over a ranked alphabet Σ is defined inductively as the smallest set of expressions st
  - $f \in T_\Sigma$ for every $f \in \Sigma_0$ and
  - $f[t_1, \ldots, t_n] \in T_\Sigma$ for every $f \in \Sigma_n$ and $t_1, \ldots, t_n \in T_\Sigma$.

- $\square$ be a special symbol of rank 0. A tree $c \in T_{\Sigma \cup \{\square\}}$ in which $\square$ occurs exactly once is a *context*, the set of all contexts over Σ is C_Σ. For $c \in C_\Sigma$ and $s \in T_\Sigma$, $c[[s]]$ denotes the tree obtained by substituting s for $\square$ in c.

Multi-dimensional trees (Rogers 2003)

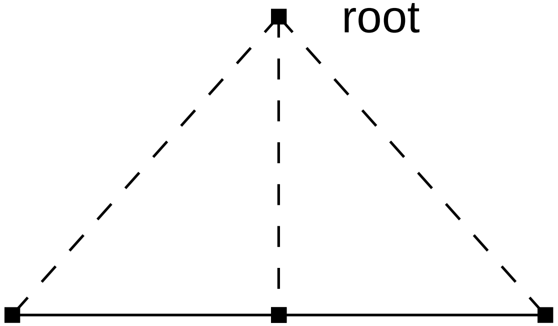
zero-dimensional tree

(point)

one-dimensional tree (string)

complex one-dimensional tree (string)

simple (local) two-dimensional tree



complex one-dimensional tree

Multi-dimensional trees (Rogers 2003)

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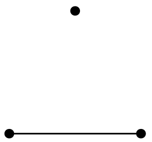
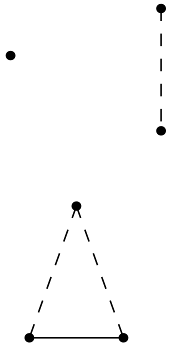
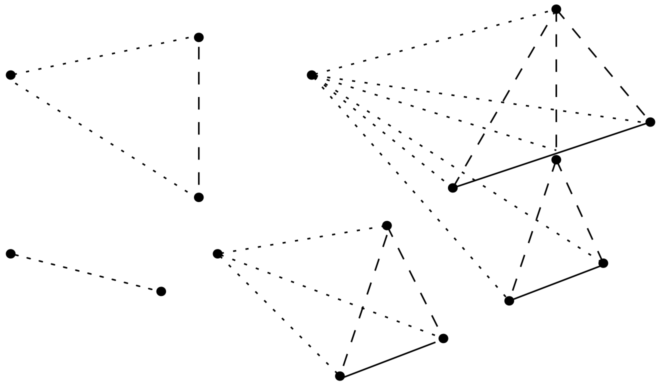
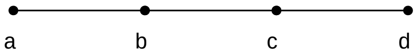
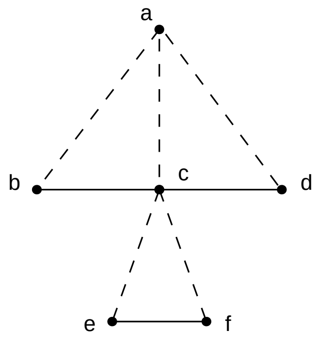
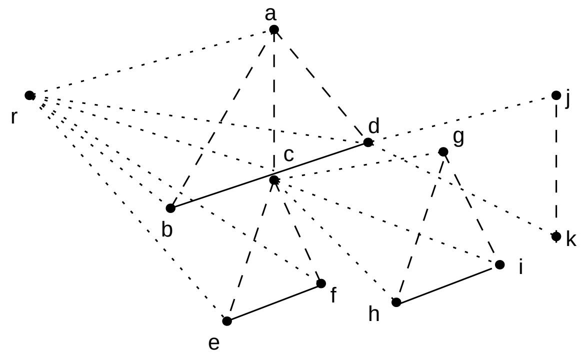
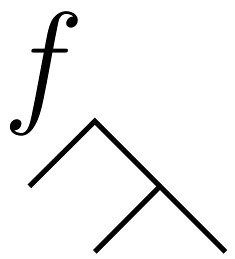
local				
composite (labeled)	-			

d-dimensional tree labeling alphabets

We will use finite d-dimensional tree labeling alphabets Σ^d where each symbol $f \in \Sigma^d$ is associated with at least one unlabeled $(d-1)$-dimensional tree t specifying the admissible child structure for a root labeled with f.



$(f \in \Sigma_t^3)$

Let Σ_t^d for $d \geq 1$ be the set of all symbols associated with t.

Multi-dimensional trees (over Σ_t^d)

Let Σ^0 be a set of constant symbols. The set $\mathbb{T}_{\Sigma^d}$ of all d-dimensional trees can be defined inductively as follows:

Definition 1 *Let ε^d be the empty d-dimensional tree. Then*

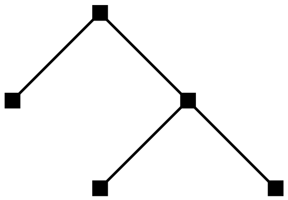
- $\mathbb{T}_{\Sigma^0} := \{\varepsilon^0\} \cup \Sigma^0$*, and*
- *for $d \geq 1$: $\mathbb{T}_{\Sigma^d}$ is the smallest set such that $\varepsilon^d \in \mathbb{T}_{\Sigma^d}$ and $f[t_1, \ldots, t_n]_t \in \mathbb{T}_{\Sigma^d}$ for every $f \in \Sigma_t^d$, n the number of nodes in t, $t_1, \ldots, t_n \in \mathbb{T}_{\Sigma^d}$ and $t_1, \ldots, t_n$ are rooted breadth-first in that order at the nodes of t.*

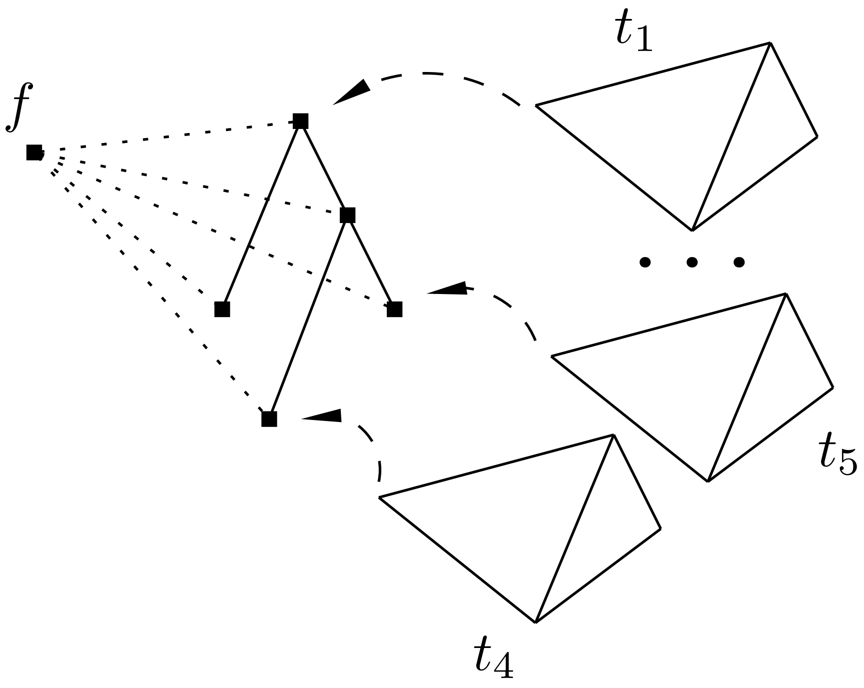
Contexts are defined as before ($\Box$ being a symbol associated with ε^{d-1}).

Multi-dimensional trees – intuition

$f \in \Sigma_t^3$

$t =$





Multi-dim. finite-state tree automata

Definition 2 *A* finite-state d-dimensional tree automaton *is a quadruple* $\mathcal{A}^d = (\Sigma^d, Q, \delta, F)$ *with*

- *input alphabet* Σ^d*,*
- *finite set of states* Q*,*
- *set of accepting states* $F \subseteq Q$ *and*
- *transition function* δ *with* $\delta(t(q_1, \ldots, q_n), f) \in Q$ *for every* $f \in \Sigma_t^d$ *where* $t(q_1, \ldots, q_n)$ *encodes the assignment of states to the nodes of* t*.*

$\delta : \mathbb{T}_{\Sigma^d} \longrightarrow Q$ *is defined such that if* $t_p = f[t_1, \ldots, t_n]_t \in \mathbb{T}_{\Sigma^d}$ *then* $\delta(t_p) = \delta(t(\delta(t_1), \ldots, \delta(t_n)), f)$*.*

Myhill-Nerode (for multi-dim. trees)

$L \subseteq \mathbb{T}_{\Sigma^d}$. For $s, s' \in \mathbb{T}_{\Sigma^d}$, $s \sim_L s'$ iff for every $c \in C_{\Sigma^d}$, $c[[s]] \in L \Leftrightarrow c[[s']] \in L$. The *index* of L is $|\{[s]_L | s \in \mathbb{T}_{\Sigma^d}\}|$.

Theorem 3 *L is regular iff L is of finite index.*

Corollary. If a tree language is of finite index, we can build an fta $\mathcal{A}_L^d$ recognizing L, with $Q = \{[s]_L | s \in \mathbb{T}_{\Sigma^d}\}$, $F = \{[s]_L | s \in L\}$, and, given some $f \in \Sigma_t^d$ and states $[s_1]_L, \ldots, [s_n]_L$, $\delta_L(t([s_1]_L, \ldots, [s_n]_L), f) = [f[s_1, \ldots, s_n]_t]_L$.

$\mathcal{A}_L^d$ is the unique minimal fta recognizing L (up to a bijective renaming of states).

Multi-dimensional trees – yield

- The (direct) yield of a d-dimensional tree is a projection on the $(d-1)$-dimensional level.
- The string yield of a d-dimensional tree can be obtained by taking the direct yield $(d-1)$ times.

Part II: Learning algorithm (multidim.)

The learner is helped by a teacher who can answer membership and equivalence queries (and return a counterexample). He maintains an *observation table*.

Definition 4 *The pair* (S, C) *(*$S \subseteq \mathbb{T}_{\Sigma^d}, C \subseteq C_{\Sigma^d}$ *finite,* $C \neq \emptyset$*) is called an* observation table *if the following holds:*

- *For every (*d*-dimensional) tree* $f[s_1, \ldots, s_n]_t \in S$*:* $s_1, \ldots, s_n \in S$ *as well –* S *is* subtree-closed, *and*
- *for every context* $c_0 \in C$ *of the form* $c[[f[s_1, \ldots, s_{i-1}, \Box, s_{i+1}, \ldots, s_n]_t]] \in C$*:* $c \in C$ *and* $s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n \in S$ *– we say that* C *is* generalization-closed*.*

Learning algorithm (multidim.)

Definition 5 *Observation table* $T = (S, C)$ *is* closed *if* $obs_T(\Sigma^d(S)) \subseteq obs_T(S)$, *and* consistent *if, for all* $f \in \Sigma_t^d$ *and all* $s_1, \ldots, s_n, s'_1, \ldots, s'_n \in S$, *if* $obs_T(s_i) = obs_T(s'_i)$ *for all* i *with* $1 \leq i \leq n$ *then* $obs_T(f[s_1, \ldots, s_n]_t) = obs_T(f[s'_1, \ldots, s'_n]_t)$.

From a closed and consistent OT $T = (S, C)$ one can synthesize an fta $\mathcal{A}_T^d$ with $Q_T = \{obs_T(s) | s \in S\}$ as set of states, $F_T = \{obs_T(s) | s \in S \cap U\}$ as set of accepting states, and $\delta_T(t(obs_T(s_1) \cdots obs_T(s_n)), f) = obs_T(f[s_1, \ldots, s_n]_t)$ for all $f \in \Sigma_t^d$ and $s_1, \ldots, s_n \in S$.

The algorithm

```
$T = (S, C)$ := $(\{a\}, \{\Box\})$ for some arbitrary $a \in \Sigma^d_{\varepsilon^{d-1}}$;
while $|\{obs_T(s) \mid s \in S\}| < I$ do
    if $T$ is not closed then $T$ := CLOSURE($T$)
    else if $T$ is not consistent then $T$ := RESOLVE($T$)
    else $T$ := EXTEND($T$)
end while;
return $\mathcal{A}^d_T$;
```

```
procedure CLOSURE($T$) where $T = (S, C)$
    find $s \in \Sigma^d(S)$ such that $obs_T(s) \notin obs_T(S)$;
    return $(S \cup \{s\}, C)$;
```

The algorithm

```
procedure RESOLVE($T$) where $T = (S, C)$
    find $c[[s]], c[[s']] \in \Sigma^d(S)$ where $s, s' \in S$ and
    $depth(c) = 1$ such that
        $obs_T(c[[s]]) \neq obs_T(c[[s']])$ and $obs_T(s) = obs_T(s')$;
    find $t, t' \in S$ such that
        $obs_T(t) = obs_T(c[[s]])$ and $obs_T(t') = obs_T(c[[s']])$;
    find $c' \in C$ such that $obs_T(t)(c') \neq obs_T(t')(c')$;
    return $(S, C \cup \{c'[[c]]\})$;
```

The algorithm

```
procedure EXTEND($T$) where $T = (S, C)$
    $\mathcal{A}_T^d$ := $synthesize(T)$;
    return EXTRACT($T, counterexample(\mathcal{A}_T^d)$);
procedure EXTRACT($T, t$) where $T = (S, C)$
    choose $c \in C_{\Sigma^d}$ and $s \in subtrees(t) \cap (\Sigma^d(S) \setminus S)$ such
      that $t = c[[s]]$;
    if there exists $s' \in S$ such that
      $obs_T(s') = obs_T(s)$ and $t \in U \Leftrightarrow c[[s']] \in U$ then
      return EXTRACT($T, c[[s']]$);
    else return $(S \cup \{s\}, C)$
    end if;
```

Conclusion

Theorem 6 *The learner returns the unique minimal automaton* $\mathcal{A}_U^d$ *for* U *(up to a bijective renaming of states) after less than* $2I$ *loop executions.*

- We have shown that the algorithm by Drewes and Högberg [2] can be used in an almost unchanged form to learn multi-dimensional trees, with the new notation.
- Consequently the algorithm is able to learn even string languages beyond the cf class, via the yield function.

Thank you!

References

[1] Angluin, D.: Learning regular sets from queries and counterexamples. Information and Computation 75(2), 87–106 (1987)

[2] Drewes, F., Högberg, J.: Learning a Regular Tree Language from a Teacher. In: Developments in Lang. Th. 2003. LNCS, vol. 2710, pp. 279–291. Springer (2003)

[3] Rogers, J.: Syntactic Structures as Multi-dimensional Trees. Research on Language and Computation 1, 265–305 (2003)

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[5] Kasprzik, A.: Making Finite-State Methods Applicable to Languages Beyond Context-Freeness via Multi-dimensional Trees. Technical Report 08-3, University of Trier. Available on: urts117.uni-trier.de/cms/index.php?id=15939
(to appear in the Post-Proceedings of FSMNLP 2008)